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ABSTRACT

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Examining Mathematics Teachers' Beliefs through Multiple Lenses

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Abstract

This paper presents a longitudinal study of an elementary mathematics teacher and describes her beliefs system in terms of Green's (1971) metaphor and provides examples of how her beliefs were enacted in her classroom practice. Dewey's (1933) notion of reflective thinking is used to explain the changes in the structure of the teacher's belief system. The purpose of the paper is to illuminate aspects of the structure of the teacher's belief system that enabled her to change some of her beliefs in a surprisingly short period of time.

Examining Mathematics Teachers' Beliefs through Multiple Lenses

> *I hate math. Math was invented by someone who was very angry as a way to get back at society. And the thought of teaching math wakes me up in the middle of the night in a cold sweat.* (Carrie's autobiography, April, 1994)

> *Teaching math is nothing more than exploring math <u>with</u> your students. I've learned that "wrong answers" are such a gift in the classroom because they open the doors for so much more understanding and exploration of math.* (Carrie's autobiography, January, 1995)

It is difficult to imagine that both of these quotes came from the same preservice elementary teacher only nine months apart. The research literature suggests that a person's beliefs cannot be changed in a short period of time, such as the time of teacher education program. Carrie's statements above call into question this long-held view about the stability of beliefs. In this paper, I present data that show both the content of Carrie's beliefs (*what* she believes) and the structure of her beliefs (*how* she believes). I trace Carrie's development over a four-year period, from her first mathematics methods course until her second year of teaching in order to illuminate aspects of the structure of her belief system that enabled her to change some of her beliefs in a surprisingly short period of time. I also examine the implications for both research and teacher education of looking at both the content and structure of a person's beliefs rather than looking only at the content of beliefs, which has been the focus of many prior studies.

Theoretical Framework

To explain Carrie's seemingly miraculous transformation, I draw on Green's metaphor for belief systems (1971) to describe the unique aspects of Carrie's belief system that seem to have enabled such change. I also use Dewey's work (1933) on reflective thinking to explain how Carrie was able to transform her beliefs within Green's structure.

Beliefs

Green (1971) provided a comprehensive description of the structure of belief systems and their properties. This description has to do with how beliefs are held rather than what those beliefs are. He elaborated on three aspects of the structure of belief systems–the relationship between beliefs, the strength with which beliefs and are held, and the manner in which beliefs are clustered. A person's beliefs can be organized by their logical order into those which are primary and those which are derived from other beliefs. Primary beliefs are so basic to a person's way of operating that she cannot give a reason for holding those beliefs; they are essentially self-evident to that person. In contrast, derivative beliefs are logically related to other beliefs. That is, when asked to provide a reason for a derivative belief, a person will tend to provide another belief as the reason.

Beliefs can also be described by the psychological strength with which they are held. Beliefs that are held with "passionate conviction" (p. 53) are called core beliefs and reside at the very center of a person's belief system. Core beliefs are not easily amenable to change and are generally fundamental to one's personality. Beliefs that are held with less psychological strength are called peripheral beliefs. Green pictured beliefs as a set of concentric circles with a core belief residing in the innermost circle and peripheral beliefs residing in the larger circles. The farther a belief is from the center, the less strongly it is held. Beliefs that are held less strongly are more open to change through examination and discussion.

A third aspect of the structure of belief systems is the way in which beliefs cluster. People tend to hold beliefs in isolated clusters so that the beliefs within a cluster are consistent, but beliefs are not necessarily consistent from cluster to cluster. Therefore, it is possible for a person to hold conflicting beliefs, but as long as they are held in isolated clusters and never placed side-by-side the person does not feel any conflict.

Green also provided a description of how people hold beliefs that is independent of their structure. He distinguished between beliefs that are held on the basis of evidence and those that are held non-evidentially. Beliefs that are held on the basis of evidence are open to criticism and modification because the reasons for the beliefs can be questioned through the presentation of additional evidence. Beliefs that are held non-evidentially,

however, are resistant to change because they are not based on reason or evidence. Therefore, it is difficult to change them through rational argument.

Green contended that the purpose of teaching is to modify students' belief systems. He argued that

> teaching is an activity aimed at the formation of belief systems having four principal characteristics:
>
> 1. A minimum number of core beliefs
> 2. A minimum number of belief clusters with a maximum number of relations between them
> 3. A maximum proportion of evidential beliefs
> 4. A maximum correspondence between the quasi-logical order of beliefs and the actual logical relations between them. (p. 52)

Green believed that these four characteristics constituted an ideal belief system. Thus, extending Green's claim about the goal of teaching to teacher education, the goal of teacher education is to help preservice teachers develop an ideal belief system with regard to the teaching and learning of mathematics. Note that Green's description of belief systems applies only to the *structure*, not the *content* of belief systems. Therefore, the goal of teacher education is not to indoctrinate preservice teachers to believe certain things; rather, the goal is to help them develop "structurally sound" belief systems so that their beliefs are evidentially held, consistent, and amenable to change.

Many people have studied the beliefs of preservice and inservice mathematics teachers. (See Thompson, 1992 for a thorough review of this literature.) What seems to elude many researchers, however, is an explanation of how teachers modify their beliefs. I posit that it is by going through a process of reflective thinking as described by Dewey (1933) that individuals are able to modify their beliefs toward a system that is more consistent with that which Green described as ideal.

Reflective Thinking

As early as 1904 John Dewey advocated the development of reflective thinking in preservice teachers. He contended that the primary purpose of teacher education programs should be to help preservice teachers reflect on problems of practice (Dewey, 1904/1965). He argued that teachers who are proficient in the skills of teaching but who lack an inquiring mind will have their professional growth curtailed. He further claimed

that teachers' lack of reflective thinking leads to intellectual dependency on "those persons who give them clear-cut and definite instructions as to just how to teach this or that" (p. 152). Thus, Dewey argued that teachers need to be helped to develop habits of reflection so that they can "break through the mesh and coil of circumstance" (p. 152) to address problems in education.

In his 1933 book *How We Think*, Dewey (1933) defined reflective thinking as "active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusions to which it tends" (p. 9). By thinking reflectively, a person can "transform a situation in which there is experienced obscurity, doubt, conflict, disturbance of some sort, into a situation that is clear, coherent, settled, harmonious" (pp. 100-101).

Dewey (1933) proposed five phases of reflective thought. When a person encounters a problematic situation, the tendency is to continue to act in the situation and to try to employ whatever suggestions for solutions come to mind. Reflective thinking requires that one suspend action to search for solutions and evaluate them. Thus, the first phase involves the recognition of possible solutions to the problem. The second phase is to problematize the situation. Dewey noted that problems do not occur in a vacuum, so the act of problematizing or intellectualizing a situation involves consciously recognizing the conditions that come to bear on the problem.

Once the problem situation has been identified, the reflective thinker enters the third phase and begins to generate hypotheses that may lead to solutions. These hypotheses are treated tentatively, as working hypotheses, and more data are gathered to refine some hypotheses and eliminate others. In the fourth phase of reflective thinking, the thinker uses reasoning to determine whether or not the hypotheses are viable solutions to the problem. Both prior knowledge and the specific circumstances of the current problem are brought to bear during this phase. After the hypotheses have been fully developed, they are tested in the fifth and final phase. This testing involves some overt action on the part of the thinker. The hypotheses are no longer used merely in a thought experiment; rather, they are actually applied in the context of the problem. At this point, the thinker can determine whether the empirical application of the hypothesis matches the

theoretical results achieved through reasoning, and either the problem is solved or the reflective process begins again.

Dewey (1933) also identified three attributes that are necessary for reflective thinking–openmindedness, wholeheartedness, and responsibility. Open-mindedness implies a freedom from prejudice and a willingness to consider new problems and entertain new ideas. Open-mindedness is characterized by the active desire to consider alternative interpretations and to examine one's beliefs, no matter how tightly they are held. Those who are open-minded are willing to leave the rut of the path of least resistance in order to pursue what may be in direct conflict with previously unexamined beliefs. Open-mindedness is not a passive quality. Open-mindedness is distinguished from empty-mindedness by the active desire to consider alternative interpretations and to examine one's beliefs, no matter how tightly they are held. Whole-heartedness involves total absorption in the problem at hand. When a person is completely immersed in a problem, questions and suggestions flow freely, and they are eagerly pursued. The attribute of responsibility is necessary to reflective thinking because it is what allows the thinker to pursue hypotheses and solutions. Intellectual responsibility requires consistency and harmony between actions and beliefs. A responsible individual carefully considers the consequences of each action and is willing to adopt the consequences when they are deemed to follow reasonably from prior conclusions. Without the requirement of responsibility, actions can be taken whose consequences are in direct conflict with beliefs.

Connections Between Green and Dewey

I propose that the way in which one makes changes to the structure of one's beliefs is by engaging in reflective thinking. By recognizing, analyzing, and resolving the problems that arise when beliefs are inconsistent, people modify their belief structures to be more in line with Green's description of an idea belief system. The crux of this process, however, is that one must be aware of inconsistencies in one's beliefs in order to act to change them. (it is worth noting, from a research standpoint, that it is the *holder* who must recognize the inconsistency. For the researcher to recognize it is insufficient.) In Dewey's terms, the situation must be seen as problematic. Posner (1982) suggested

that researchers have found little change in people's beliefs over time because people are generally unaware of inconsistencies in their beliefs, so they do not recognize problematic situations as they arise.

Cooney, Shealy, and Arvold (1998), in their study of preservice secondary mathematics teachers, coined the term *reflective connectionist* to describe a person who "integrates voices, analyzes the merits of various positions, and comes to terms with what he or she believes in a committed way" (p. 330). In a subsequent article Cooney (1999) elaborated on the definition of a reflective connectionist, noting that reflective connectionists recognize inconsistencies in their own beliefs, critically examine new information for its compatibility with their belief structures, and are amenable to changing their beliefs in order to deal with new information. Cooney et al. noted that reflective connectionists have the ability to use knowledge in an adaptive, reflective way, and they claimed that when change occurs through thoughtful reflection it has the potential to be generative because it lays the foundation for the careful examination of new ideas in the future. Thus, although Cooney et al. did not draw specifically on the work of Dewey, it seems that their description of a reflective connectionist fits with my claim that changes in belief structures occur when people engage in reflective thinking.

Methods

This paper is based on data collected during two studies, the first of which I refer to as the Initial Study and the second of which I refer to as the Follow-up Study. During the fall of 1994 I conducted a study of four preservice elementary teachers who were enrolled in their first mathematics methods course. Carrie, who is the focus of this paper, was one of the four preservice teachers in this course. The four preservice teachers constituted a special section of the course for which I was the instructor. The course was field-based with most of the time spent in a fourth grade classroom observing mathematics instruction, conducting task-based interviews with individual children, and teaching small groups. There was also extensive time for dialogue between the preservice teachers, the mentor teacher, and me. The purpose of the initial study was to document the sense-making practices of the preservice teachers with regard to mathematics

teaching and learning by noting issues that they found problematic and describing the means by which they resolved these problematic issues.

I kept anecdotal records of informal conversations with Carrie during the first two years of her teaching career. During the first year, Carrie was employed as a teaching assistant in a first grade classroom in the school where she completed her student teaching. The second year she obtained employment as a second grade teacher in this same school. During the third year, I conducted the follow-up study to examine Carrie's mathematics teaching practices. The purpose of this study was to document the manifestation of Carrie's beliefs in her mathematics teaching practice.

Data Collection

Data collected during the initial study included Carrie's written autobiography (at the beginning and end of the study), two individual interviews, Carrie's journal, four audiotapes of Carrie conducting task-based interviews with individual children, three audiotapes of Carrie teaching a small group, one videotape of her teaching a small group, my field notes on eight observations of the classroom teacher, and audiotapes and field notes from eight discussions among the four preservice teachers, the mentor teacher, and me. All of the data collection in this study was done by me. See Author (1999) for more details of the study.

Data from the follow up study included field notes from weekly classroom observations throughout the school year, two individual interviews, and classroom artifacts such as lesson plans, classroom displays, and student work. Half of the data collection in this study was done by a graduate research assistant, and half of it was done by me.

Data Analysis

I analyzed the data using Green and Dewey's frameworks. Initially, I attempted to ferret out Carrie's beliefs and their structure by identifying statements that Carrie made and the rationale for the statements. The rationale that Carrie gave helped me determine if beliefs were primary or derivative and whether they were held evidentially or non-evidentially. As I developed a hypothesis about the structure of Carrie's beliefs, I

attempted to fit new data into this structure. Where the data did not fit, I revised the structure or noted the data as disconfirming evidence for the structure I had created. Once I had developed a robust picture of Carrie's beliefs and their structure, I searched the data looking for compelling and illustrative examples for each belief. Finally, in an effort to explain how the structure of Carrie's beliefs changed, I searched the data for evidence of Dewey's phases of reflective thinking. I noted evidence that Carrie was engaging in Dewey's phases and evidence of the attributes of reflective thinking.

Carrie's Background and Beliefs about Mathematics

Carrie's Background

Carrie was a Caucasian female, and at the time of the initial study she was a 21-year old college senior majoring in elementary education. She chose to major in elementary education because she thought that elementary children are at a crucial age where they need to be loved and cared for, and she wanted to provide that love and caring. Carrie demonstrated a strong care ethic and stated that she wanted her classroom to be a safe and loving environment for children. Prior to entering her teacher education program, Carrie had worked with preschool children through a daycare job, all ages of children through coaching swim team, and high school students through a religious organization.

When Carrie was in high school, her mother insisted that she take 4 years of mathematics so that she would not eliminate any of her options for higher education or careers. Carrie did well in mathematics in high school, earning As and Bs, but she did not enjoy the subject. In her mathematics autobiography at the beginning of her teacher education program she wrote, "Math has always been labeled as my worst subject even though I've always made good grades. My brother is a 'math whiz,' and this was always discouraging for me." She described herself as a right-brained, creative person and said that she thought of mathematics as a structured and left-brained activity.

When asked at the beginning of her preservice program to complete metaphors about mathematics, she revealed some of her frustrations with learning mathematics. She said that mathematics is like alphabet soup with numbers, referring to the fact that she

found mathematics confusing and lacking any logic. She also said that mathematics was like a headache that will not go away. This comment reflected her frustrations with having taken 4 years of high school mathematics in order to get into college, even though she disliked the subject intensely. She reported that she had always viewed mathematics as a hurdle she had to get over in order to get to something desirable, such as college. Carrie's metaphor for learning mathematics was "Learning mathematics is like putting a puzzle together and finding out that one piece is missing." She explained that she often followed intricate algorithms to solve problems but got the wrong answer because she made a careless error.

Carrie's experiences as a mathematics student led her to believe that mathematics is a dull, lifeless, illogical subject. She saw no place for personal excitement, enjoyment, or creativity in learning mathematics. Her views about mathematics contrasted sharply with her views about literature and language arts. Carrie was quite passionate about these content areas and believed that they were fun, interesting, creative, and an outlet for self-expression. This dichotomous view of mathematics and language arts has been documented with other elementary teachers (e.g., Schifter, 1996; Yaffee, 1993). Carrie was, in fact, critical of teaching practices that reduce language arts to its mechanical components and prevent students from seeing the beauty inherent in poetry or literature. For example, Carrie noted

> I think teachers do a disservice to their kids in English where you spend so many weeks picking poems apart for iambic pentameter and "What does this mean?" and "Where is the irony?" and you never understand that poetry is so beautiful because you're so sick of picking it apart and learning the English part of it. You don't just absorb it and think "Gosh, this is beautiful!" And I think we do that to kids too much.

Carrie did not, however, see that this was precisely the experience she had had with mathematics; mathematics for her had been reduced to its procedural elements, which prevented her from seeing any beauty in the subject. As a preservice teacher, Carrie thought that teaching mathematics amounted to explaining boring rules and procedures to children and giving them lots of problems for practice. Because she had never experienced the beauty, creativity, or self-expression that is possible with mathematics, she could not conceive of ways of teaching or learning mathematics that could enable her

students to have experiences that were different from her own. Clearly, Carrie's beliefs about mathematics were derived from her personal experiences and were thus held evidentially, making them amenable to change.

Carrie's Core Belief

As a beginning preservice teacher, Carrie was very concerned about the societal and family situations that impact negatively on children's lives, and her core belief stemmed from these concerns. Carrie believed that school should be a place where children interact with adults who love and care about them as human beings, and she saw school as a place that could rectify the unpleasantness in some children's lives. She expressed this strong care ethic in my first interview with her a few weeks before she began her first mathematics methods course. "It's really important for all kids, but especially little kids, to get love that I just don't think they're getting [at home]." As is characteristic of core beliefs, Carrie held this belief passionately, and it was the driving force behind her career choice.

Carrie's core belief was part of her general outlook on life and was not confined to her views about education. This belief was very closely associated with her religious convictions, a connection noted by Cooney (1999) with other preservice teachers. Carrie noted on numerous occasions that her goal in her interactions with children was to treat them with the type of love, compassion, and respect that was consistent with her religious beliefs.

> The more I study and the more that I learn, it just really breaks my heart, you know, the families that those children come from. Then just to be shoved off in the side of a classroom because you don't understand why they are the way that they are. It really hurts to know that that happens every single day and that those children aren't ever given a chance at life. Maybe that's why–I feel that's a big reason why I want to teach.

During the initial study, all of Carrie's expression of her core belief dealt with children's lives, in general, and did not connect in any way to their academic or intellectual lives. At the end of her second year of teaching, Carrie still held the same core belief, but she had expanded her thinking to include concern for children as learners.

> The most important thing to me about who I am as a teacher is–when they leave this room, I want them to know that there was someone older than them that really

> loved them and invested in their lives and cared enough to stop a lesson and get a [bandage] and thought that everything they did was great. And even when they made mistakes, that was still okay because as long as they thought about it, that was awesome.

Carrie's core belief was also a primary belief because her beliefs about teaching, learning, and children were derived from this fundamental belief about the purpose of schooling. When I asked her for reasons to support her statements about students, learning, and teaching, she returned time and again to her desire to show children that she cared for them. For the sake of semantic simplicity, I will refer to this core and primary belief as Carrie's belief in respecting children.

Cooney (1999) noted that many preservice teachers display a care ethic similar to Carrie's and that this care ethic is manifested in wanting to shield children from intellectual struggle in mathematics because that struggle is deemed unpleasant. Initially, I thought that Carrie perhaps had a propensity to want to make math fun and easy for children, thereby eliminating intellectual challenges. Some of her early lessons as a preservice teacher were, in fact, characterized by the use of manipulatives or candy in order to motivate children with less regard for the instructional appropriateness of these materials. Most of these lessons, however, involved substantive content. In my observations of Carrie's teaching during her preservice years, it was sometimes difficult to determine whether Carrie was challenging her students intellectually because she struggled mightily with classroom management, especially when manipulatives were involved. During my observations in Carrie's second year of teaching, however, I was able to gain a better sense of the role of intellectual challenge in her classroom. By and large, Carrie posed tasks that were on an appropriate level for her students.

However, on several occasions I observed lessons where the content was not immediately within the grasp of all of the children. One such lesson dealt with children trying to decide whether forty-five should be written as 405 or 45. In this lesson, approximately one-fourth of the children thought that 405 was an acceptable representation. Carrie used questioning to get other children in the class to explain why they thought 45 was the correct representation, and then asked questions to determine if students thought that 405 and 45 were the same number. For the most part, Carrie asked

questions and let the students reason with one another without any judgment from her. Some of the students became frustrated, and others who initially through that 45 was correct became confused and thought that 405 was correct. Carrie seemed comfortable letting this discussion and its ensuing confusion continue. When the students reached a stalemate in their discussion, Carrie asked questions to review what they knew about place value and how many hundreds were in forty-five and 405 and 45. At the end of this discussion, some students were still confused, and Carrie told them that it was fine if it did not all make sense right now. She assured them that they would return to this discussion on another day. From incidents like this one, I became convinced that while Carrie did want her students to have fun in mathematics class, she also placed significant value on intellectual challenge. She did not shy away from problems and questions that required her students to think and struggle.

Carrie's Derivative Beliefs

Carrie held three sets of beliefs that were derived from her primary belief about respecting children. These beliefs focused on students, learning, and teaching. In our interviews, I probed in order to get Carrie to articulate how these beliefs applied specifically to the teaching and learning of mathematics. However, through her comments and her actions in the classroom, it became apparent that Carrie's beliefs applied more broadly to all subject areas that she was responsible for teaching.

When asked to talk about her beliefs, Carrie often talked about the same ideas in multiple contexts, suggesting that these beliefs were held with great psychological strength. For example, Carrie talked about the importance of rewarding students for their thinking when she talked about her beliefs about students. She talked about the importance of having students explain their answers when she talked about her beliefs about learning. And when she talked about her beliefs about teaching, she said that she thought it was important for her to model her mathematical thinking for her students in order to get them to share that type of thinking with their peers. Thus, the notion of mathematical thinking was a salient one for Carrie and permeated her belief cluster about teaching and learning mathematics. This notion was clearly tied to her core belief about respecting children and their ways of thinking.

Another idea that came up in all three contexts was the notion of dealing with incorrect answers. In the context of her beliefs about students, Carrie said that it was important to find something of value in a child's thinking, even if the thinking led to an incorrect answer so as not to diminish the child's self-confidence and discourage further participation. When talking about learning, Carrie noted that it was important for children to see value in wrong answers because they provide a venue for further learning. And in the context of talking about teaching, Carrie said that she thought it was important for her to admit to making mistakes in front of her students so that they would see that being wrong was not a tragedy. The value of incorrect answers was salient for Carrie and was derived from her core belief about the importance of respecting children and their ideas –even if they are not completely correct.

In the sections that follow, I provide some evidence of how Carrie's beliefs were manifested in her classroom practice in order to show that the beliefs that Carrie professed were not simply beliefs that she articulated verbally. They were beliefs that governed her classroom practice. Because the same practice provided evidence for multiple beliefs, as noted above, I give examples from Carrie's classroom practice in only one context to avoid redundancy.

Beliefs about students. Carrie believed that children must have confidence in themselves as learners if they are to be successful in school and in life. Closely related to this belief was the importance that she placed on children showing respect for one another and for their thinking. She noted

> The most important thing is that a kid can do anything and can solve anything if they feel confident. If you boost their confidence and believe in them, and help them to believe in themselves, then even if they never get the right answer for a story problem or any kind of a math problem, they still will be okay with continuing to try.

Carrie's belief about students was derived from her core belief about respecting children because in her rationale for this belief, Carrie referred to her desire to make children feel intellectually safe in her classroom. She said that she wanted every student to be comfortable taking intellectual risks in her classroom. Carrie's beliefs about students were manifested in her teaching practice in the way that she handled incorrect answers, and the way that she assessed student learning.

Carrie's views on the value of incorrect answers in mathematics class changed dramatically during her professional education program. Initially, she saw mathematics as cut-and-dried with answers that were either right or wrong, and wrong answers were to be avoided because they reflected badly on the person who provided them. This view was likely derived from her own experiences and frustrations with learning mathematics. By the end of her first mathematics methods course, however, Carrie said, "I've learned that wrong answers are such a gift in the classroom because they open the doors for so much more understanding and exploration of math." During her first mathematics methods course, Carrie had the opportunity to observe a teacher who valued the process of learning and treated wrong answers as learning opportunities. Seeing a teacher who handled wrong answers this way seemed to provide Carrie with a model of how to deal with wrong answers while still respecting children's thinking and feelings. She was able to integrate this practice into her teaching style because it supported her core belief in respecting children. During her second year of teaching, Carrie expressed her views by saying

> As long as you are thinking, it is okay to get the wrong answer because nobody gets all the right answers in life all the time. You are not going to be right all the time, and as long as it's in an environment where you are not like, "Go change; this is the wrong answer!" or kids laugh at you for getting the wrong answer. I think it's really as much of a learning process as the right answer is–even more so sometimes.

Carrie thought it was important to find something of value in each child's thinking, regardless of the correctness of the response. For example, during a lesson on place value and renaming numbers, Carrie asked the students how many ways they could write 82. The first child volunteered the numeral 82, and the second child responded with the written word "eighty-two." The third child said "two tens and eight ones." Carrie asked him to come to the board and write the number. He wrote 28 and then shook his head "no." Carrie asked him what he was thinking, and he said that it was not 82 because it was backwards. Carrie then praised him for catching his error and noted "Sometimes you have to see it to be sure, don't you?"

The next child said that 82 could be represented by 8 + 2. Carrie asked her to come to the board and explain her thinking, and the child wrote 8 + 2 = 82. Carrie asked

her orally if "eight plus two is eighty-two," and the child said, "No, it's 10!" Carrie then said, "OK, now how can you use that information to revise your answer? What is special about the 8 in 82? What does it stand for?" The child was quickly able to correct her answer to say 80 + 2 = 82 without any embarrassment.

On other occasions when children displayed errors in their thinking, Carrie would say, "Not quite, but I see what you are thinking." Then she would ask the child a question to help uncover and correct the error. Carrie was very adept at asking children questions that helped them clarify their thinking without allowing them to be ashamed, and this practice stemmed from her belief that it was important to value and reward children's thinking, regardless of its correctness, in order to boost their self-confidence. She was concerned that if children became discouraged by giving incorrect answers, they would disengage from the learning process, so she tried to avoid this discouragement as much as possible.

Carrie's assessment practices also stemmed from her belief about the importance of showing respect for children and their thinking. She thought it was important to assess each child's understanding of a topic and individualize instruction as much as possible. She explained how she planned for lessons: "The most important thing is knowing where each kid is in that skill, and that really determines for me what we are going to do that day–to really be able to gear it to what they need." Carrie used whole class instruction with the children seated on a rug, which she called "caucus time," for the majority of each lesson. Most lessons ended with individual seat work, pair work, or work in small groups. She used her questions during caucus time to asses each child's level of thinking about the topic at hand. She also adjusted her questions to an appropriate level for different students. During seat work, pair work, or small group work, she individualized assignments and provided individualized assistance to challenge some students and to provide background help for others.

Carrie said that she rarely used pencil and paper assessments, preferring instead to interview children "to see if they have the concept." The school system in which Carrie taught used a grading system of E for excellent work, G for good work, S for satisfactory work, and U for unsatisfactory work. Carrie noted that she sometimes gave children an "E" for showing improvement rather than for performing at an excellent level for second

grade. She tended to reward effort as well as, and in some cases more highly than, performance.

Beliefs about learning. Carrie believed that learning is a process of understanding, not a means to a correct answer. This belief was also derived from her core belief in the importance of respecting children because Carrie reflected her desire to create an intellectually safe environment in her classroom when asked to justify her belief about learning. Carrie manifested this belief in her teaching practice by insisting that children explain their answers and by seeking and rewarding multiple solution processes.

Carrie routinely asked students to explain how they arrived at their answers. This request usually resulted in procedural answers from children, and Carrie sometimes asked the child to extend the explanation to address conceptual aspects of the problem. For example, when working on two-digit addition problems that involved regrouping, the children often explained their thinking in terms of "3 plus 9 is 12 so I wrote down the 2 and put the 1 over here." Carrie would ask why the child did not put the 12 in the answer to encourage the child to explain how place value was important in the problem.

Carrie frequently asked students "Who thought of it another way?" and then gave several children opportunities to explain their thinking. For example, when solving the problem 6 + 9, she allowed children to explain three solutions–counting on from first, counting on from larger, and making 10. She continually rewarded children for using different solutions. Sometimes, this practice encouraged children to give nonsense answers because they were "different" ways of thinking about the problem. For example, during the aforementioned lesson on place value, when Carrie asked for multiple ways of saying 82 one child said, "HB because H is the eighth letter of the alphabet and B is the second letter of the alphabet." Carrie praised the child's creativity but did not direct the class to stick to numerical explanations. Therefore, for each subsequent number that Carrie posed, a child used the "alphabet naming method." Carrie was not bothered by this addition to her lesson and allowed the children to continue, but she did not spend any more time on validating the alphabet method.

Carrie sometimes posed problems that had more than one correct answer. One lesson involved small groups of children sorting their shoes by some attribute and then allowing other groups to guess what attribute was used. Carrie encouraged the students to

think of as many different ways that the shoes could be sorted as possible. The group that had done the sorting was generally intent upon the "guessers" figuring out their strategy, but Carrie praised any plausible solution. She said, "Is that the one [attribute] you used? No? But does it make sense? Good! That's another way to think about it!"

Carrie summarized her thoughts about learning as a process this way:

> It's not really about math per say; it's about teaching kids to think for themselves and teaching them that their ideas are valuable and they have great ways of figuring things out. Even if it's not the way that you would figure it out, which is so neat that it Carries on to every subject that you teach.

Beliefs about teaching. Carrie believed that teachers should be role models for their students–both role models of learners and of good citizens. This belief was derived from her core belief about respecting students because Carrie said that she thought that in order for her students to feel intellectually safe in her classroom, she had to be willing to engage in the same types of learning that she expected of them–explaining their thinking and learning from incorrect answers. Carrie's belief about teachers as role models manifested itself in the mathematics classroom by modeling mathematical thinking for students, admitting to making errors, and admitting to not knowing an answer.

In order for her students to be willing to engage in the types of discussions she desired, Carrie thought that it was imperative that she model this type of thinking for them. She said, "A lot of it is telling them how I came up with an answer and them thinking about how many different ways there are to come up with that answer." For example, during a lesson on measurement the students were given gourds, balance scales, and metric weights and were to determine how much the gourds weighed. Carrie demonstrated with one gourd before dismissing the students to their groups. She placed the gourd in the scale and showed the students the various weights she could place in the opposite pan–20 g, 10 g, 5 g, and 1 g. She asked the students what weight she should start with, and many students said 1 g. She started putting 1 g weights in the pan, and the scales did not move. She stopped and told the class that she was thinking that it would take a very long time to get the pans to balance using 1 g weights. She held the gourd in one hand and a 1 g weight in the other and told the students that the gourd was much heavier than the 1g weight. Then the students suggested that she start with the 20 g

weights. She began adding 20 g weights to the pan and had the class count along with her by 20s until the pan began to move. Then she asked the students what weight to try next. They proceeded to add weights from largest to smallest to the pan until the scale balanced.

Some of the students were having difficulty counting by 20s and then 10s, 5s, and 1s. When they were finished balancing the scale, Carrie said, "You know, I was thinking that there might be another way to do this, and it might be even easier than the way we just did it. It makes sense to start with the 20 g weight because it is the heaviest, but sometimes it can be hard to count by 20s. I think it is easier to count by 10s, so I'm going to do it again, this time starting with the 10 g weights." The second time, Carrie used only 10 g and 1g weights, and the students were able to count along easily because this counting mirrored what they had been doing with place value. Carrie frequently modeled both her mathematical thinking and her pedagogical thinking for her students, as this example demonstrates.

Carrie believed that it was important to "be human" and admit to making errors or admit to not knowing the answer in front of her students. She said, "I guess by letting them know that I am human and that I make mistakes, makes it a little easier [for them] to make mistakes. When I get excited when they get the wrong answer, then they feel okay about finding an answer that is wrong." I did not observe Carrie making any unintentional mathematical errors while she was teaching; I did observe her making intentional errors and allowing the children to explain what was wrong with her thinking. In other instances Carrie showed students that she was unsure about something such as vocabulary (e.g., the appropriate use of the words "die" and "dice"), spelling of mathematical terms (e.g., parallelogram), and origins of mathematical terms (e.g., why we call 60 seconds one minute). Other times, Carrie incorrectly interpreted students' explanations of their mathematical thinking. For example, a student explained her solution to the problem 15 – 6 using a counting back strategy. When Carrie tried to restate the student's thinking for the rest of the class, she used a missing addend strategy. Carrie asked the child if this was what she meant, and the child said no. So Carrie apologized to the child and told the class that she had misunderstood the child's

explanation. She asked everyone to listen intently as the child explained her solution strategy again.

Carrie said that she was very comfortable with students asking her questions for which she did not know the answer. She gave as an example a child in her class whose parents were both university mathematics professors. The child regularly investigated mathematics at home with his parents and came to school wanting to share his newfound knowledge with his teacher. He even took some delight in trying to stump Carrie with problems that he had figured out at home with his parents. One day, the child made an icosahedron with plastic linking shapes and asked Carrie what the polyhedron would be called. Carrie was not certain whether the polyhedron had a formal name, and if it did, she could not remember it. So, she worked with the child to list everything they knew about the figure (e.g., solid figure, 20 triangular faces). Then they decided how they could use that information to help them determine the name for the object (e.g., look it up in a book, ask someone). Carrie said that it was important for the child to know that she was proud of him for investigating problems on his own and for him to know that she was interested in what he was doing and willing to work with him on problems. She had no qualms about telling the child that she did not know the answer to something, and, in fact, she thought that it was valuable for the child to see a teacher genuinely struggle with a mathematical problem.

Summary. Carrie's beliefs about students, learning, and teaching were derived from her core belief about respecting children because when asked to explain these beliefs, she continually referred back to her desire to create an intellectually safe classroom environment where children felt valued. These beliefs were psychologically central for Carrie because she held them with passion and was willing to go to great lengths to affirm these beliefs in her teaching practice. There were no apparent inconsistencies among these beliefs, suggesting that they were held in the same cluster. Carrie's derivative beliefs were held evidentially, based on her experiences working with children and her own experiences as a learner. Carrie summarized her beliefs about children, teaching, and learning this way

> I feel like a year of school is so much more than do you just…can you spit back the concepts. Have you learned how to be a better friend this year? Have you learned how to be loving? Have you learned how to work out problems? Have

> you learned how to not tell and try to figure it out yourself. I mean, have you learned how to use everything you know to make a hypothesis? Have you learned how to be a better person by the end of the year?

Changes in Carrie's Beliefs

Carrie's beliefs about mathematics teaching changed dramatically during her preservice program. Initially, she disliked mathematics intensely and did not want to teach it. But, by the end of her teacher education program, she was actually looking forward to teaching mathematics. As the classroom excepts above suggest, Carrie was able to enact her new beliefs in her classroom teaching practice. In the next section of this paper, I describe the manner in which these changes occurred. To describe the changes exhibited by Carrie, I draw on Dewey's work on reflective thinking (1933) to describe the process by which Carrie changed. As she engaged in the reflective process, the structure of her belief system changed to be more in line with what Greene (1971) described as ideal–a minimum number of core beliefs, a minimum number of clusters of beliefs with maximum relationship between the clusters, a maximum proportion of evidentially held beliefs, and a maximum correspondence between the quasi-logical order of beliefs and their actual logical order.

Reflective Thinking

Carrie possessed the three attributes that Dewey identified as necessary to reflective thinking: open-mindedness, wholeheartedness, and responsibility. Despite her previous experiences with mathematics, Carrie was extraordinarily open-minded with regard to teaching mathematics. She genuinely wanted to find a way to teach mathematics that would be consistent with her other beliefs so she actively sought alternatives, and she embraced evidence that mathematics could be different from the way she experienced it. Carrie exhibited whole-heartedness from my first interview with her. She immediately articulated her concerns about teaching mathematics and her desire to find a resolution to the conflicts she felt. Carrie was whole-hearted in her willingness to seek advice, to try new teaching strategies, and to engage in reflective conversations about her mathematics teaching with others. She was very analytical in her pursuit of a teaching style that was consistent with her beliefs, and she was very committed to trying

to enact this style in her classroom. Carrie manifested the attribute of responsibility when she recognized the disharmony between her beliefs about children, teaching and learning and her beliefs about mathematics. She was troubled by this conflict, stated it openly, and actively sought to resolve the conflict. From the outset of the initial study, Carrie saw herself as having the responsibility to make sense of the complicated enterprise of teaching. She believed that it was imperative that she gather as much evidence from as many source as possible (e.g., prior personal experiences, field experience, teacher education courses) and analyze this information in order to act on it in a way that made sense to her.

We can trace Carrie's progress through Dewey's phases of reflection and the resulting impact on her beliefs and practices. As a preservice teacher, Carrie did not look forward to teaching mathematics to children, mainly because she did not think she could do a good job of it. Carrie's beliefs about mathematics and herself as a learner and teacher of mathematics were in sharp contrast to her other beliefs, in particular, her belief about teachers as role models. Carrie's belief that a teacher is a role model was a source of significant conflict for her because she knew that she was not able to model excitement and enthusiasm for learning mathematics. Dewey argued that in order to find a satisfactory resolution to a problematic situation, one must suspend action and recognize possible solutions to the problem. As a preservice teacher, Carrie saw one optimal solution to the problem of teaching mathematics. Initially, her solution was to decide that she would not make a good mathematics teacher. Thus, she absolved herself of any responsibility for rectifying the situation. She was essentially saying that the belief clusters were in conflict and the belief cluster about respecting students was more important. Thus, the solution to the conflict was not to teach mathematics. However, it is the rare elementary teacher who does not need to teach mathematics, either as a discipline in itself or in the service of another discipline such as science or social studies. Carrie's proposed solution was not very pragmatic.

The second step in Dewey's reflective process is to problematize the situation and identify various conditions that bear upon it. Carrie identified the problem in these words:

> I would never want for a child to grow up with a thing about math–with a negative attitude or anything, and I would just be petrified that I would give that to them. So I would rather not teach them math and have another teacher teach

> them math than give them a stigma about math....I just don't think that I could teach it with the enthusiasm that I could teach English with or any other art or anything.

The next phase of Dewey's reflective thinking process is generating hypotheses, and it was at this stage that Carrie hit a dead-end. There were no hypotheses to generate about the problem as Carrie defined it because choosing not to teach mathematics was not really something that was in her control. Further, during her teacher education program she was going to have to take two mathematics methods courses, whether she planned to teach mathematics or not. So, Carrie returned to the first step of the reflective thinking process and generated a new solution to her problem. The new solution was that she could try to learn to teach mathematics in a way that was consistent with her other beliefs about teaching and learning. Thus, the problem now was to learn to genuinely enjoy teaching mathematics so that she could provide a positive learning experience for her students. Carrie articulated the problem this way:

> ...since I have such a negative thing about math, but I really don't want to give that to my students. It will be neat to learn creative and exciting ways to teach them math and get them to love it more than I ever could.

Carrie hypothesized that she could learn to teach mathematics in way that was consistent with her other beliefs about teaching and learning by "working with a great teacher that really loves what she's doing." Carrie reasoned that the only way that she knew how to teach mathematics was based on the models that she had seen from her teachers. She recalled her experiences with learning mathematics by saying, "Math was math. You couldn't really do much with math....It was more like punishment than education. [I need] more experience in math because I have such as negative attitude toward it." Clearly, Carrie was actively seeking opportunities to see that teaching mathematics could be different than the way she had experienced it as a student.

Carrie had an opportunity to test her hypothesis that she could learn to teach mathematics differently by participating in a 10-week mathematics methods course that was primarily based in a fourth-grade classroom. (See Author, 2000 for more details about the design of the field experience.) The classroom teacher was an exemplary and enthusiastic mathematics teacher. At the end of this methods course, Carrie saw that it

was possible for students to experience mathematics in the same way she experienced language arts–as a dynamic, creative, fun, interesting discipline with opportunities for individual exploration and interpretation. She was able to find ways to teach that were consistent with her other beliefs.

When Carrie became a teacher, she had an opportunity to further test her hypotheses about teaching mathematics. She found that she was able to teach mathematics in ways that supported her central cluster of beliefs by making mathematics interesting and rewarding for students. She was able to emphasize the process of learning and was able to be a good role model for her students. Across the years of her teacher education program and her first few years of teaching, Carrie continued to reflect on teaching mathematics in order to strike an appropriate balance between fun and creative lessons and lessons that contained significant academic content. As time went on, she managed to find a way to teach mathematics that suited her personality and belief systems and that resulted in meaningful learning for her students. During her second year of teaching Carrie said, "Isn't it amazing that I used to hate math and now it's my favorite subject to teach?" For Carrie, the problem of teaching mathematics was resolved. She had brought her conflicting belief clusters into line and had resolved the conflict.

Structure of Carrie's Beliefs

The structure of Carrie's belief system seems to be very close to what Green described as ideal. Green's first criterion for an ideal belief system was that a person should hold a minimum number of core beliefs. Carrie appeared to hold one core belief about the importance of respecting children. She held this belief with passion and ferocity, and it was the driving force in her teaching. Green noted that when one holds a core belief with such passion that it must be affirmed at all costs, it enables one to open other beliefs for scrutiny and inquiry. Such was the case with Carrie. She so passionately held her belief about respecting children that she was constantly scrutinizing her other beliefs (or, perhaps more accurately, their manifestation in their teaching practice) in order to be sure that they supported her belief in respecting children.

Green's second criterion for an ideal belief system was that it should contain a minimum number of clusters with a maximum number of relationships between the

clusters. Carrie's beliefs about students, learning, and teaching were derived from her core belief about respecting children, so there were strong connections among the beliefs in this cluster. Her cluster of beliefs about mathematics initially had no connections to this central cluster. However, as she changed her views about school mathematics, she was able to make connections between her central cluster of beliefs and her beliefs about school mathematics. Her beliefs about mathematics as a discipline and about herself as a learner of mathematics remained unconnected to her central cluster of beliefs, however.

The third criterion suggested by Green was that there should be a maximum proportion of evidentially held beliefs. Carrie's beliefs about mathematics were held evidentially; they were based on her past experiences as a learner of mathematics. Thus, these beliefs were amenable to change because she was willing to consider evidence that contradicted her previous experience. What Carrie took as evidence was strongly influenced by her central cluster of beliefs. In order for her to be convinced that she could be an effective mathematics teacher, she had to see evidence that mathematics could be taught in a way that did not undermine students' self-confidence and in a way that valued the process as much as the product. Once Carrie saw a role model for this type of teaching during her field experience, she was willing to try to shape her mathematics teaching to be consistent with her beliefs. When she was successful teaching mathematics in a way that did not conflict with her central belief cluster, she was willing to reevaluate and modify her beliefs about school mathematics.

The last criterion for an idea belief system offered by Green was that there should be a correspondence between the quasi-logical order of beliefs and the objective logical order. I interpret this to mean that the beliefs that one claims are derived from other beliefs must be supported by evidence. In the preceding sections I have established that Carrie's beliefs about students, learning, and teaching were logically derived from her core belief about respecting children. Further, I have suggested that her beliefs about mathematics were not initially logically related to her central cluster of beliefs, but after being presented with relevant evidence, Carrie was able to connect some of her beliefs about mathematics to her central cluster of beliefs.

Discussion and Implications

Carrie's story calls into question the long-standing assumption (backed by much empirical evidence) that beliefs are not easily amendable to change. Carrie's case suggests that within some parameters, beliefs can be changed rather expediently. Looking at both the content and structure of Carrie's beliefs helps to explain this apparent contradiction with earlier literature.

Looking only at the content of Carrie's beliefs provides an incomplete picture. Through the lens of the content of Carrie's beliefs, we initially see someone with beliefs about mathematics that match much of what is already reported in the literature. This perspective provides us with little new information from a research perspective and with no viable avenues for changing beliefs from a teaching perspective. If we look at Carrie's beliefs at the end of her second year of teaching, we see someone with beliefs that appear to be more in line with what the mathematics education community would like to see in teachers. However, this view gives us little useful information toward understanding what enabled the change in her beliefs. It seems virtually impossible to look at the structure of someone's beliefs without looking at the content as well. In order to determine the structure of beliefs, it is necessary to determine which beliefs are derived from others and which are isolated. To do this requires addressing the content of the beliefs.

Looking at both content and structure together provides a robust picture that furnishes some leverage for change from both a teaching and a research perspective. From a teaching standpoint, we may find more inroads to help preservice teachers become aware of and challenge their beliefs. In Carrie's case, her beliefs about students, teaching, and learning were so strongly held that she had to affirm them at any cost. Thus, she required herself to resolve the inconsistencies between these beliefs and her beliefs about mathematics. Perhaps as teacher educators, we need to take a broader view of our students and their beliefs and try to understand not just their beliefs about mathematics but also their wider beliefs about education, human relationships, and a person's role in society. Indeed, Pajares (1992) suggested that beliefs substructures must be examined in light of the more general, and perhaps more central, beliefs that a person holds. By doing so, we may gain insights into how a person's beliefs about mathematics are connected to a larger set of beliefs that a person holds. We may also gain insight into what kinds of evidence are salient to a particular person. If what one takes as evidence is

logically related to the evidence that would support or defy a belief, then it seems plausible that those beliefs can change. If, however, what one takes as evidence is not logically related to one's beliefs, then there seems to be little hope for changing beliefs. By examining what counts as evidence in a wider sphere of beliefs, we may be able to challenge preservice teachers' beliefs with different forms of evidence than we have considered previously. We may also be in a better position to see potential perturbations that might prompt preservice teachers to reexamine their beliefs if we look at beliefs about mathematics as situated in a larger context. And from a research standpoint, we may gain greater access to preservice teachers' beliefs by approaching them through the lens of other beliefs.

Carrie was somewhat unique in that she came to her teacher education program aware of her internal inconsistencies and searching for answers. She exhibited Dewey's attributes of openmindedness, wholeheartedness, and responsibility, and she was predisposed to think critically about her beliefs and practices. (Similar cases have been reported by Yaffee, 1993 and Schifter, 1996.) I question whether I would have known this about Carrie if she had been merely a student in my methods course and not a participant in a research study. I wonder how many other students like Carrie I have not seen in my classrooms because *I* was not openminded and listening for evidence that they wanted to see mathematics differently. It would have been easy to dismiss Carrie based on what came to the surface initially. However, the structure of Carrie's belief system made her a prime candidate for the type of change that we hope will occur as part of a teacher education program. How future teachers hold their beliefs may be more important than what they believe (Cooney, 1999). Again, looking at preservice teachers' beliefs in a broader context may help us become aware of more Carries in our classes.

Not all students are intrinsically motivated like Carrie. Many students do not come to us aware of their conflicting views and seeking a resolution to some inner turmoil. Perhaps it should be a goal of a mathematics methods course to raise preservice teachers' awareness of tensions among their beliefs. Indeed, Thompson (1992) claimed that teacher educators should strive to help teachers uncover for themselves their beliefs and practices and any inconsistencies in them. She argued that teacher educators cannot simply present alternative ways of acting and believing to teachers and expect them to

adopt these new stances. She noted that "for teachers, intrinsic motivation for considering alternatives must come from their own experiences in the classroom" (p. 143). Similarly, Simon & Schifter (1991) highlighted a contrast between professional development activities that help teachers acquire new teaching strategies and those that provide teachers with opportunities to experience fundamental shifts in their views of teaching and learning. Clearly, the potential for significant change lies in the latter because the latter leads to generative change. If teachers simply learn new activities and strategies for teaching, they are not necessarily empowered to transfer these new techniques to other topics in mathematics or to other situations. In contrast, if teachers experience a fundamental shift in their beliefs and are aware of this shift, then they have the potential to continue to grow and change as they encounter new situations.

Carrie's story provides convincing evidence that a teacher education program and teaching experience can have a positive impact on one's beliefs and instructional practice. The challenge that remains for teacher educators is how to facilitate this type of transformation for other students.

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